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Translated by R.C.G.

PMM U.S.S.R.,Vol.52,No.6,pp.746-753,1988
Printed in Great Britain

0021-8928/88 $10.00+0.00

THE ASYMPTOTIC FORM OF THE STATIONARY SEPARATED CIRCUMFLUENCE OF A BODY AT HIGH REYNOLDS NUMBERS*

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An asymptotic theory of the stationary separated circumfluence of bodies at high Reynolds numbers, Re, is constructed. It is shown that the length and width of the separated zone (SZ) is proportional to Re and that the drag cofficient is proportional to Re^{-1}. A cyclic boundary layer is located around the separated zone with a constant vorticity. In the scale of the body, the flow tends towards a Kirchhoff flow with a velocity on a free line of flow of the order of $\mathrm{Re}^{-1/2}$ which satisfies the Brillouin-Villat condition.

A review of the attempts which have been made to describe the two-dimensional separated circumfluence of a body at high Reynolds numbers is given in /1, 2/. Certain features of the asmyptotic structure of the solution based on qualitative arguments were pointed out in /3, 4/. The corresponding shape of the separated zone was calculated in /5/. However, no complete theory was constructed in these papers. The appearance of the numerical calculations in /6, 7/ stimulated further investigations and a model with a non-zero jump in the Bernoulli constant on the boundary of the separated zone was proposed in /8/. A number of hypotheses concerning the limiting structure of the flow were put forward in /9/.

In the solution obtained below the flow in the scale of the body is described as in /1, 2/ but the velocity is of the order of $\mathrm{Re}^{-1/4}$. The flow characteristics in this zone are correspondingly renormalized. The flow in the scale of the separated zone corresponds to the assumptions made in /3, 4/. Unlike in /1-4/, the flow in the scale of the body is not directly combined with the flow in the scale of the separated zone. There are several embedded zones and the possibility of uniting these ensures the selfconsistency of the expansion. Moreover, the cyclic boundary layer on the boundary of the separated zone plays an important role.

1. Let us transform to dimensionless variables by employing the characteristic size of the body and the velocity at infinity as the scales. As $\mathrm{Re}\to\infty$, let the length and width of the separated zone tend to infinity while remaining of the same order. Then, in the limit, the flow in the scale of the separated zone will be a vortex potential flow /10/. According to the Prandtl-Batchelor theorem, the vorticity is constant in the detached domains (Fig.1).

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*Prikl.Matem.Mekhan.,52,6,958-966,1988

Outside of these zones, the flow is potential. There may be a tangential discontinuity on the boundary of the vortex zones. The jump, Δ, in the Bernoulli constant on this discontinuity must be determined from an analysis of the cyclic boundary layer. This layer develops along the discontinuity from the point A to the point B, turns round in the neighbourhood of B and moves from B to A along the axis of symmetry. Close to A, it again turns round and, on becoming adjacent to the approach stream, again moves towards B. The velocity profile in the boundary layer does not change during these rotations. The corresponding boundary value problem has been posed in /11/.

Let us introduce the longitudinal and transverse coordinates s and n (Fig.1) in the cyclic boundary layer and let us denote, by $U(s)$ the velocity of the vortex potential flow on the internal boundary. In the case of the function $g = {}^1/_2(u^2 - U^2) - \Delta$, where u is the velocity along the s-axis in the boundary layer, the boundary value problem in Mieses' variables is:

$$\frac{\partial g}{\partial s} = \frac{u}{\mathrm{Re}} \frac{\partial^2 g}{\partial \psi^2} \tag{1.1}$$

$$s = 0,\ \psi < 0,\ g = 0 \tag{1.2}$$

$$\psi > 0,\ g(0, \psi) = g(s_A, \psi) \tag{1.3}$$

$$\psi = 0,\ s_B < s < s_A,\ \partial g/\partial\psi = 0 \tag{1.4}$$

$$\psi \to +\infty,\ g \to -\Delta = \mathrm{const} \tag{1.5}$$

(ψ is the stream function). The variables are normalized on the initial scales buy only the leading terms as $\mathrm{Re} \to \infty$ are retained in the equation. An analysis of the properties of the boundary value problem (1.1)-(1.4) provides us with grounds for supposing /12/ that its bounded solution as $\psi \to \infty$ is unique, while condition (1.5) enables us to determine Δ. A proof of this uniqueness is given below for the actual case being considered. The solution itself is obvious: $g \equiv 0$. Consequently, $\Delta = 0$ also. This means that the model in /8/ is wrong.

Theorem 1. The system of Eqs.(1.1)-(1.4) with the additional condition that $g \to \mathrm{const}$ as $\psi \to +\infty$ has a unique solution $g = 0$, $\mathrm{const} = 0$.

Proof. Let us suppose that the opposite is true. The properties of the function $u(s, \psi)$ which are necessary for the general theorems used in the proof to be applicable follow from the physical meaning of the problem under consideration. In (1.1), we shall subsequently assume that $u > 0$ and that it is an arbitrary, smooth, bounded and known function. Then, Eq.(1.1) is linear with respect to g and, without any loss in generality, it may be assumed that $g \to 1$ as $\psi \to +\infty$.

Let us prove that $g(s, \psi) \leqslant 1$. We shall supplement the definition of $g(s, \psi)$ when $s_B < s < s_A$ for $\psi < 0$ by putting $g(s, -\psi) = g(s, \psi)$, $u(s, -\psi) = u(s, \psi)$.

Let s and ψ exist such that $g(s, \psi) > 1$. Then, $g(s, \psi)$ attains an absolute maximum at a certain point s_m, ψ_m. By virtue of (1.3), it may be assumed that $s_m > 0$ without any loss of generality. Then, by selecting a sufficiently small $\delta > 0$ and a sufficiently large M, we obtain that $s_m - \delta \leqslant s \leqslant s_m$ in the domain $|\psi| \leqslant M$, which contradicts the general principle of a maximum (Theorem 1 from Sect.1 in /13/). Consequently, $g \leqslant 1$.

Let us prove that $g(s, \psi) < 1$. In order to do this, we repeat the previous argument, making use of the fact that $g(s, \psi) \leqslant 1$ and the reinforced principle of a maximum (Theorem 6 from Sect.1 in /13/). By assuming that $g(s, \psi) = 1$ at a certain point, we obtain that $g \equiv 1$, which constradicts the boundary condition $g(0, \psi) = 0$ when $\psi < 0$.

Let us now consider the domain $\psi \geqslant N > 0$. According to what has been proved, $g(s, N) < 1$ and it follows from the continuity of $g(s, N)$ that there exists an $\varepsilon > 0$ such that $g(s, N) < 1 - \varepsilon$. When $\psi \geqslant N$, by virtue of (1.3), the function $g(s, \psi)$ can be periodically extended with respect to s on the whole of s. When $\psi \geqslant N$, let us consider a function $y(s, \psi)$ which is such that $y(s, \psi)$ satisfies Eq.(1.1), $y(0, \psi) > g(0, \psi)$, $y(s, N) = 1 - \varepsilon$, $y(0, \psi) \to \mathrm{const}$ when $\psi \to \infty$.

When $s > 0$, the function $y(s, \psi)$ dominates $g(s, \psi)$: $y \geqslant g$. Let us show that $y(s, \psi) \to 1 - \varepsilon$ when $s \to \infty$, $\psi = \mathrm{const}$. We put $y(s, \psi) = 1 - \varepsilon + y_1(s, \psi)$ and extend $y_1(s, \psi)$ unevenly into the domain $\psi < N$: $y_1(s, \psi - N) = -y_1(s, N - \psi)$. Then, $y_1(s, \psi)$ will be a solution of the equation $y_{1s}' = u_1 y_{1\psi\psi}''$, where $u_1 = u$ when $\psi \geqslant N$ and u_1 is an even function with respect to N.

Then,

$$\frac{dI}{ds} = 2\int_{-\infty}^{+\infty} y_{1\psi}' (u_1 y_{1\psi\psi}'')_\psi' \, d\psi = -\int_{-\infty}^{+\infty} u_1 (y_{1\psi\psi}'')^2 \, d\psi < 0$$

$$I(s) = \int_{-\infty}^{+\infty} (y_{1\psi}')^2 \, d\psi$$

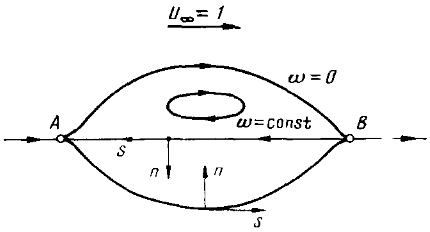

Fig.1

Consequently, the function $I(s) > 0$ decreases monotonically and this means that it tends to a certain limit. Hence, $y''_{1\psi\psi} \to 0$ when $s \to \infty$ and $y_1 \to k_1\psi + k_2$. However, the function $y_1(s, \psi)$ is bounded. This means that $k_1 = 0$ and, since $y_1(s, N) = 0$, then $y_1 \to 0$. Now, from the inequality $g(s, \psi) \leqslant y(s, \psi)$ when $s \to \infty$, we obtain $g(s, \psi) \leqslant 1 - \varepsilon$. This contradicts the initial assumption that $g(s, \psi) \to 1$ when $\psi \to +\infty$.

Hence, the vortex potential flow which is obtained at the limit does not have a discontinuity in the Bernoulli constant. Such a solution is not uniformly suitable close to the lines where there is a discontinuity in the vorticity.

Let us consider the boundary layer on the boundary of the separated zone. The leading term in the velocity expansion in this layer is equal to the velocity of the non-viscous flow $U(s)$. The leading vorticity term satisfies the equation

$$\frac{\partial \omega}{\partial s} = \frac{U(s)}{\mathrm{Re}} \frac{\partial^2 \omega}{\partial \psi^2} \tag{1.6}$$

Moreover,

$$\psi = U(s)\, n \tag{1.7}$$

within the layer apart from higher-order terms.

The boundary conditions for Eq.(1.6) are

$$\begin{aligned} & s = 0,\ \psi < 0,\ \omega = -[H]\,\delta(\psi - 0) \\ & \psi > 0,\ \omega(0, \psi) = \omega(s_A, \psi) \\ & \psi = 0,\ s_B < s < s_A,\ \omega(s, 0) = 0 \\ & \psi \to \infty,\ \omega \to \omega_\infty \end{aligned} \tag{1.8}$$

Here, use has been made of the fact that, by virtue of the effectively non-viscous nature of the flow during the rotation of the boundary layer, the profile $\omega(\psi)$ does not change. The contact within the domain of rotation between the approach stream and the rotating boundary layer leads to the appearance of a discontinuity $[H]$ in the Bernoulli constant in this region. The δ-function in (1.8) is the vorticity distribution which corresponds to the velocity discontinuity since $\partial H/\partial\psi = -\omega$, where $H = u^2/2 + p$ and p is the pressure in the boundary layer. Problem (1.6), (1.8), in a general formulation, has been studied in /12/*. (*The formula for the replacement of the variables at the start of Sect.2 in /12/ must have the form $\psi = 2p\sqrt{t_A}$, $t_A = t(s_A)$. The values of $E(\tau)$ and not 100 $E(\tau)$ are shown in Table 2 in /12/.) It follows from the results in /12/ that its solution only exists subject to the condition

$$\omega_\infty = -2D_0[H]\sqrt{\mathrm{Re}/t(s_A)}$$
$$\left(t(s) = \int_0^s U(s)\,ds, \quad D_0 = D(b, 0), \quad b = \frac{t(s_B)}{t(s_A)}\right)$$

(the function $D(b, \xi)$ has been tabulated in /12/). $\omega_\infty > 0$ and $[H] < 0$ in the lower half of the separated zone which is considered. The circulation of the velocity of the vortex potential flow around the lower half of the separated zone, $t(s_A) = \omega_\infty S/2$, where S is the overall area of both halves of the separated zone. Hence,

$$\omega_\infty = -2[H]\,D_0\sqrt{2\mathrm{Re}/\omega_\infty S} \tag{1.9}$$

By calculating the cyclic boundary layer, it is possible to make use of the effectively non-viscous nature of the flow in the domain where the stream reunites and to calculate the remote trail and the drag of the body. By replacing the independent variable ω by $h = H - H_\infty$, where H_∞ is the Bernoulli constant in the approach stream and integrating the equation for $h(s, \psi)$ over the whole of the cyclic layer, it can be shown that the drag coefficient (which is calculated over twice the characteristic dimension) is equal to

$$c_D = \omega_\infty^2 S/\mathrm{Re} = C/\mathrm{Re} \tag{1.10}$$

Since the magnitude of S is proportional to the square of the length and is inversely proportional to the length of the separated zone, we have

$$C = \omega_\infty^2 S = \mathrm{const} \tag{1.11}$$

2. Although the flow in the scale of the body does not directly combine with the flow in

the scale of the separated zone, we shall pass immediately to its description as this enables us to determine the principal characteristics of the flow. (The subsequent analysis of the intermediate domains only confirms the selfconsistency of the model). Let us assume that, in the limit, the flow in these domains is effectively non-viscous. This assumption will be confirmed after the velocity scales have been determined. The tangential discontinuity, which according to what has been previously said, occurs at the beginning of the separated zone, must be extended into all of these domains up to the body. The jump in the Bernoulli constant on this discontinuity is constant. Hence, in the limit, the flow in the scale of the body will be a flow according to the Kirchhoff scheme with the same discontinuity in the Bernoulli constant, $[H]$. (Ho $[H] \neq \Delta = 0$. These are the parameters of the flows in the difference characteristic scales). The velocity at infinity in the body scale is then equal to $V_{1\infty} = \sqrt{-2[H]}$. The drag coefficient of the body $c_D = k_D V_{1\infty}^2 = -2k_D[H]$, where k_D is the Kirchhoff drag coefficient. By comparing this with (1.10), we obtain

$$-[H] = C/(2k_D \mathrm{Re}) \tag{2.1}$$

The Reynolds number, calculated using $V_{1\infty}$ and the dimensions of the body: $\mathrm{Re}_1 = \mathrm{Re}\, V_{1\infty} = (C\,\mathrm{Re}/k_D)^{1/2} \to \infty$. Hence, the assumption regarding the non-viscous nature of the flow in the body scale and in the limit is confirmed. The flow in this scale has been described in detail in /1/ but the velocity and the Reynolds number must be renormalized in accordance with the scales indicated above. The limiting flow satisfies the Brillouin-Villat condition /1/ which uniquely determines k_D.

It follows from (1.9), (1.11) and (2.1) that $\omega_\infty = 2CD_0^2 k_D^{-2}\mathrm{Re}^{-1}$, $S^{1/2} = k_D^{-2}\mathrm{Re}\,C^{-1/2}D_0^{-2}/2$. Then, the length of the separated zone is equal to

$$L = \frac{k_D^2\,\mathrm{Re}}{2\alpha^{1/2}C^{1/2}D_0^2}, \quad \alpha = \frac{S}{L^2} = \mathrm{const} \tag{2.2}$$

3. As has already been noted, the resulting structure of the flow is similar to that indicated in /3, 4, 14, 15/. However, no combining of the solutions in the different characteristic domains was carried out in these papers and even the question as to whether they could be combined was not posed. This leads to a difference in the formulation of the problem concerning the boundary layer on the boundary of the separated zone. The methods of determining the discontinuity in the Bernoulli constant turn out to be fundamentally different. Above, relationship (1.9) between $[H]$, ω_∞ and the dimensions of the separated zone is a condition for a solution of the boundary value problem exists for a cyclic boundary layer which satisfies the conditions for its combination with a non-viscous flow. A similar link was obtained in /14, 15/ from energy considerations.

Let us now consider the rate of energy dissipation in greater detail. Let us instal a reference surface at a considerable distance from the body. According to Bobylev's formula (see /16/, for example), the rate of energy dissipation within the reference volume is equal to

$$\int_V \Phi\, dv = \frac{1}{\mathrm{Re}} \int_V \omega^2\, dv + \frac{2}{\mathrm{Re}} \int_{\partial V} (\mathbf{u}\nabla\mathbf{u})\,\mathbf{n}\, ds$$

(the rate of energy dissipation is normalized using the characteristic scales). It can be shown that the second integral on the right tends to zero as the boundary of the reference volume tends to infinity. It follows from this that the magnitude of c_D is determined by expression (1.10). This suggests that the problem concerning the cyclic layer has been correctly formulated. At the same time, the condition that the drag coefficients calculated from the parameters in the remote trail and from the rate of energy dissipation should be the same is identically satisfied and it cannot be used as a closure relationship.

This, however, was, in fact, done in /15/ where, instead of a cyclic boundary layer, it was postulated that there is a layer resulting from the mixing of two streams with a discontinuity in the Bernoulli constant $[H]$ which finds itself under the action of a longitudinal pressure gradient. In this case, the thickness of the loss of momentum in the trail depends on $[H]$. The equality between the thickness of the loss of momentum in the trail and the loss of momentum corresponding to the rate of energy dissipation within the separated zone served in /15/ to determine $[H]$. Moreover, in /14/ the opinion was expressed that the rate of energy dissipation outside of the separated zone corresponds to a "second dissipative layer" and is not therefore taken into account.

The main weakness of this reasoning is the replacement of a cyclic layer by a mixing layer and only taking account of the rate of energy dissipation within the separated zone in calculating the thickness of the loss of momentum in the trail.

So, the orders of the length and width of the separated zone, the drag coefficient of the body and the limting state of the flow in the scale of the separated zone were correctly, although not rigorously, predicted in /3, 4/. In this respect, the model in /3, 4, 14, 15/

does correspond to the results in the present paper. The method used for the closure of the models, the number of characteristic domains, the structure of the boundary layer on the boundary of the separated zone and the quantitative results are different. Unlike in /3, 4, 14, 15/, the selfconsistency of the asymptotic form which has been constructed is shown in this paper.

4. The constants C, α and b are defined by the solution of the problem of a vortex potential flow without a discontinuity in the Bernoulli constant /5/.

The most complete data have been presented in /17/. The divergence in the results noted in these papers when the discontinuity in the Bernoulli constant $\Delta \to 0$ and $\Delta = 0$ was not confirmed in a later paper /18/, the data in which for $\Delta = 0$ are identical with the results in the preceding papers when $\Delta \to 0$.

As a result of precessing the data in /5, 17, 18/, one obtains $\alpha \approx 0.443$, $C = \omega_\infty^2 S \approx 74.9$, and $b \approx 0.545$. The ratio of the halfwidth of the separated zone, W, to its length is equal to 0.300. Interpolating the data from /12/ and carrying out the calculation we again obtain $D_0 \approx$ 0.235, which is identical within the limits of the assumed accuracy (1%). In the case of a circular cylinder, $k_D \approx 0.50$. Thus, in the case of a circular cylinder

$$c_D = 74.9\ \mathrm{Re}^{-1} \tag{4.1}$$

$$L = 0.393\ \mathrm{Re} \tag{4.2}$$

$$\omega_\infty = 33.1\ \mathrm{Re}^{-1} \tag{4.3}$$

$$[H] = -74.9\ \mathrm{Re}^{-1} \tag{4.4}$$

$$W = 0.118\ \mathrm{Re} \tag{4.5}$$

Here, the radius has been used as the scale of length while the magnitude of c_D was calculated from the diameter of the cylinder.

One of the principal effects which are described by the theory is the retardation of the stream in the scale of the body under the influence of the separated zone. However, according to (4.4), when $\mathrm{Re} \approx 150$, the velocity at infinity in the body scale is equal to unity, that is, this effect completely disappears. It is therefore difficult to anticipate that there will be quantitative agreement between theory and the results of numerical calculations at the Reynolds number which are attained in /7/ ($\mathrm{Re} \leqslant 300$). However, although the nature of the dependence of many flow parameters on the Reynolds number changes at $\mathrm{Re} \approx 150$, according to the calculation, the length of the separated zone continues to increase linearly at the same rate.

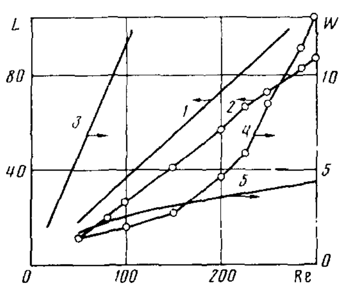

Fig.2

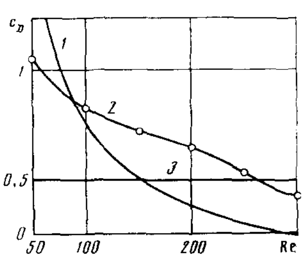

Fig.3

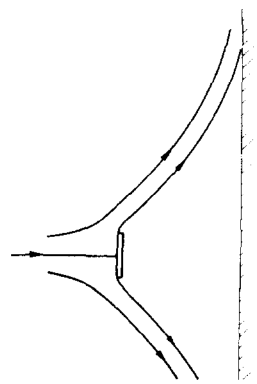

Fig.4

In Fig.2, the straight line *1* corresponds to (4.2), the broken line *2* corresponds to the numerical calculation of the length of the separated zone /7/, the straight line *3* corresponds to formula (4.5), the broken line *4* represents the numerical calculation of the half-width of the separated zone in /7/ and line *5* corresponds to the model in /2/ ($W = 0.25\ \mathrm{Re}^{1/2}$). According to the model /2/, the length of the separated zone in the notation adopted here is given by the formula $L = 0.39\mathrm{Re}$ which is identical to (4.2) within the limits of accuracy of the calculation. It is seen that, when $\mathrm{Re} < 150$, the half-width of the separated zone corresponds to those predicted in /2/ and that it is only when $\mathrm{Re} > 150$ that it begins to approach the asmyptotic value. The transition from the behaviour corresponding to the model in /2/ to the asymptotic behaviour described here is also characteristic of the other quantities. According to the model in /2/, $c_D \to 0.50$ as $\mathrm{Re} \to \infty$. The numerical results (Fig.3 in which *1* represents the results from (4.1), *2* represents the results from /7/ and 3 represents the results from /2/) demonstrates the tendency for c_D to decrease below 0.50 when $\mathrm{Re} \approx 250$. The results in this paper also correspond to a reduction in the magnitude of the maximum in the vorticity on the surface of the body as Re is increased from 250 to 300 and the other features of the flow at the upper limit of the Reynolds numbers which are attainable in numerical calculations.

5. Let us now consider the domain of rotation of the boundary layer. Since the scale of the separated layer is proportional to Re, the thickness of the boundary layer is of the order of unity. The boundary layer equations are no longer valid close to points A and B (Fig. 1). As point A or point B is approached, the thickness of the boundary layer behaves as $U(s)^{-1}$ (see (1.7)). Therefore, the characteristic scale of the turning region $l_4 \sim U^{-1}(l_4)$. It follows from the results in /19/ that $U(s) \sim (s/\mathrm{Re}) \ln (s/\mathrm{Re})$. (The number Re appears here because the size of the separated zone is proporitonal to Re). Consequently, $l_4 \sim \sqrt{\mathrm{Re}/\ln \mathrm{Re}}$. The velocity scale in this domain $U_4 \sim l_4^{-1} \sim \sqrt{\ln \mathrm{Re}/\mathrm{Re}}$ and the characteristic Reynolds number $\mathrm{Re}_4 = U_4 l_4 \mathrm{Re} = \mathrm{Re}$. It follows from these estimates that the flow is effectively non-viscous in the domain under consideration.

Furthermore, since $[H]/U_4^2 \sim 1/\ln \mathrm{Re}$ and $\omega_4 l_4/U_4 \sim 1/\ln \mathrm{Re}$, the flow is potential in the leading term in this region. The following term in the expansion is just $\ln \mathrm{Re}$ times smaller than the leading term and must be taken into account both when there is vorticity and when there is a discontinuity in the Bernoulli constant. The main term, a potential flow with a critical point, has a complex potential $w_4 = C_1 (z/l_4)^2$, $C_1 = \mathrm{const}$. Here, $z = x + iy$ where x and y are Cartesian coordinates. The external flow has the asymptotic form /19/

$$z/\mathrm{Re} \to 0, \quad \omega_5 \sim C_2 (z/\mathrm{Re})^2 \ln (z/\mathrm{Re})\, \mathrm{Re}$$

The overlap region of these expansions is quite small. Similar situations have been discussed in detail in /20/. In complete analogy with /20/, w_4 unites with w_5 when $C_1 = -C_2/2$ in the intermediate limit when $z \sim \mathrm{Re}^{1/2}$.

The following is important in the analysis as a whole: a non-contradictory description of the turning zone exists and the flow in this domain is effectively non-viscous. We further note that there is no discontinuity in the Bernoulli constant in the recombination region. Hence, the characteristic "projection" which is traced so well in the results in /7/ in the rear part of the separated zone does not arise here.

6. The expansion which has been constructed above is not uniformly applicable close to the point A at which the square of the flow velocity is comparable with $[H]$ with respect to its order of magnitude which must lead to the occurrence of an unusual projection. The characteristic dimension of the corresponding region, l_3, and the velocity scale U_3 are connected by the relationship $U_3 \sim U_4 (l_3/l_4)^2 \sim (-[H])^{1/2} \sim \mathrm{Re}^{-1/2}$, whence

$$l_3 \sim l_4 (\ln \mathrm{Re})^{-1/2} \sim \mathrm{Re}^{1/2} (\ln \mathrm{Re})^{-1}$$

Since $\mathrm{Re}_3 = \mathrm{Re}\, U_3 l_3 = \mathrm{Re} (\ln \mathrm{Re})^{-1} \to \infty$ in this domain, the flow here is effectively non-viscous. The scale of the stream function $\Psi_3 \sim (\ln \mathrm{Re})^{-3/4}$. By virtue of the condition $\omega = 0$ in the cyclic layer on the axis of symmetry, $\omega(\psi) \sim \mathrm{const}\, \psi\, \mathrm{Re}^{-1}$ as $\psi \to 0$. Hence, the scale of the vortex in the domain being considered $\Omega_3 \sim \Psi_3 \mathrm{Re}^{-1} = \mathrm{Re}^{-1} (\ln \mathrm{Re})^{-1}$. Since $\Omega_3 l_3 = \mathrm{Re}^{1/2} (\ln \mathrm{Re})^{-2} \ll \mathrm{Re}^{1/2} \sim U_3$, the flow in the main approximation in this region is also potential. As this flow contains the line of discontinuity in the Bernoulli constant, it involves the impingement of two potential flows with different Bernoulli constants. Since the potential of this flow tends to $\mathrm{const}\, (z/l_3)^2$ as $z/l_3 \to \infty$, the possibility of its merging with the exterior region is obvious.

7. It is impossible to achieve coalescence of the flow in the scales of the projection and the body since, in the body scale, the separated zone expands in proportion to $x^{1/2}$ while a flow of the type of Chaplygin flow /21/, which occurs in the scale of the projection, is proportional to $x^{3/2}$. Hence, in order to complete the proof of the non-contradictory nature of the expansion, it is necessary to show that an intermediate zone exists which can coalesce with the Kirchhoff and Chaplygin flows. The velocity scale is the same in all three zones and is determined by the magnitude of $[H]$. Consequently, the flow in the main approximation is effectively non-viscous.

Let us now consider the separated potential circumfluence of a plate which is parallel to the wall (Fig.4) and make the ratio of the length of the plate to the distance between the plate and the wall tend to zero. The flow in the scale of the plate tends to a Kirchhoff flow while, in the scale of the distance to the wall, it tends to a Chaplygin flow. The complex velocity of the flow being considered is equal to

$$\frac{dw}{dz} = \left(1 - \left(\frac{1}{2} + i\left(h^2 - \left(\frac{1}{w} - \left(h^2 + \frac{1}{4}\right)^{1/2}\right)^2\right)^{1/2}\right)^{-1}\right)^{1/2}$$

All the stream lines of this flow in Fig.4 were obtained numerically.

The distance from the plate to the wall is of the order of the parameter $h \to \infty$, the length of the plate $\sim h^{-3}$ while the size of the transition zone, the existence of which can be directly demonstrated, is $\sim h^{-1}$. Since there is no fundamental difference between the splicing of the flows in the scales of the body and the projection and the splicing of the domains of the flow which has just been considered, the existence of an intermediate domain is proved. Its longitudinal dimension is of the order of $l_3^{1/2}$. The width of the separated zone is of the order of $l_3^{1/4}$. The proof of the non-contradictory nature of the theory which has been constructed is now complete.

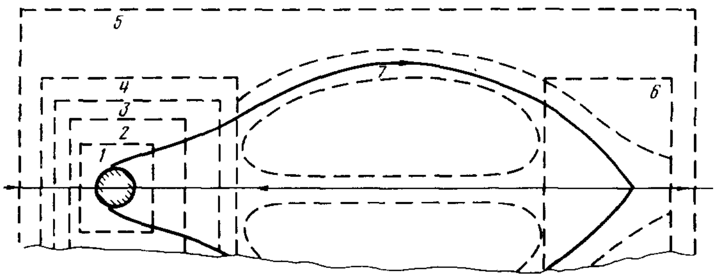

Fig.5

8. The structure of the flow as a whole is shown in Fig.5. Here *1* is the domain of Kirchhoff flow, *2* is the intermediate domain (paragraph 7), *3* is the domain where the potential flows impinge upon one another (paragraph 6), *4* and *6* are the regions where the cyclic layer rotates (paragraph 5), *5* is the domain of vortex potential flow and *7* is the cyclic boundary layer.

We note that the coalescence in the main term of the flows in the body scale, and in the scales of the intermediate domain, the region where the flow rotates and the domain where the flow is potential does not enable us to relate the dimensions of the body and the separated zone, since the flow in the intermediate domain can be constructed for any ratio of the dimensions of the body and the scale of the projection only if the latter is much greater than the dimensions of the body. The condition that the drag coefficients calculated from the flow parameters in the body scale and from the parameters in the remote trail should be the same (formula (2.1)) was made use of above for the closure of the system of relationships. This condition cannot be obtained from the joining of the different domains as joining is possible when the condition is violated. Hence, the model which can be obtained by solely considering the leading terms turns out to contain an arbitrary parameter. The standard method of determinining this parameter involves the construction of the higher terms of the expansion of the solution: the condition for their existence must enable us to remove the arbitrariness in the selection of the leading term.

In many cases, similar difficulties can be successfully circumvented by establishing a certain property of the solution and requiring that it should be preserved on passing to the limit. Actually, the well-known Prandtl-Batchelor theorem on the constancy of the vorticity in the domain of the closed stream lines is proved in this manner. The condition that the drag coefficient calculated using the different methods should be constant was employed as such a condition in deriving (2.1). Hence, from a formal point of view, (2.1) must be considered as the condition for the problem concerning the higher approximations to be solvable.

It is known that the drag coefficient of a plate set up parallel to the direction of flow is due to the force of friction and is of the order of $\mathrm{Re}^{-1/2}$, that is, it is greater than in the case of a plate which is perpendicular to the flow. Let us now consider the frictional force during the symmetric circumfluence of a wedge. As the aperture angle of the wedge is reduced, the Kirchhoff drag coefficient, k_D decreases. At the same time, in accordance with (2.4), the dimensions of the separated zone also decrease while, in accordance with (2.1), the velocity in the body scale and, together with it, the frictional force increase. Hence, the results of the theory which has been described above cannot be considered as contradicting the well-known results for a plate which is parallel to the flow. The construction of a unified theory which describes the transition from one type of flow to another is an exceedingly complex problem.

The author thanks G.Yu. Stepanov and G.G. Chernyi for discussing this paper.

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Translated by E.L.S.